On the quantum analogue of Galileo's leaning tower experiment

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Abstract.

The quantum analogue of Galileo's leaning tower experiment is revisited using wave packets evolving under the gravitational potential. We first calculate the position detection probabilities for particles projected upwards against gravity around the classical turning point and also around the point of initial projection, which exhibit mass dependence at both these points. We then compute the mean arrival time of freely falling particles using the quantum probability current, which also turns out to be mass dependent. The mass dependence of both the position detection probabilities and the mean arrival time vanish in the limit of large mass. Thus, compatibility between the weak equivalence principle and quantum mechanics is recovered in the macroscopic limit of the latter.

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1. Introduction

As a consequence of the equality of gravitational and inertial mass, all classical test bodies fall with an equal acceleration independently of their mass or constituent in a gravitational field. Historically, the first experimental study to probe this feature was conceived by Galileo with test bodies in free fall from the leaning tower of Pisa[1]. In modern times several tests have been performed with pendula or torsion balances leading to extremely accurate confirmations of the equality of gravitational and inertial masses[2]. Though most of these schemes consider only classical test bodies, there exist indications about the validity of the equality of gravitational and intertial masses even for quantum mechanical particles using the gravity-induced interference experiments[3, 4]. The universal character of the law of gravitation, however, has a much richer structure than the above equality, as embodied in the principle of equivalence in its various versions.

There are three statements of the equivalence principle which are equivalent according to classical physics but are logically distinct. Holland[5] emphasized the importance of separating them clearly in order to discuss their quantum analogues: (i) *Inertial mass is equal to Gravitational mass;* $m_i = m_g = m$. As mentioned earlier, the compatibility of this equality with quantum mechanics has been verified in several experiments[3, 4]. (ii) *With respect to the mechanical motion of particles, a state of rest in a sufficiently weak, homogeneous gravitational field is physically indistinguishable from a state of uniform acceleration in a gravity-free space.* A natural quantum analogue of this statement is[6]: "The laws of physics are the same in a frame with gravitational potential $V = -mgz$ as in a corresponding frame lacking this potential but having a uniform acceleration g instead". This can verified in quantum mechanics by transforming the Schrödinger wave function for a quantum particle in a gravitational potential to that in an accelerated frame lacking this potential[7]. Predictions of the Schrödinger equation in a noninertial frame have been shown to be experimentally observed[6]. (iii) *All sufficiently small test bodies fall freely with an equal acceleration independently of their mass or constituent in a gravitational field.* To obtain its quantum analogue this statement might be replaced by some principle such as the following[5]: "The results of experiments in an external potential comprising just a (sufficiently weak, homogeneous) gravitational field, as determined by the wavefunction, are independent of the mass of the system". The status of this last version of the equivalence principle for quantum mechanical entities is the subject of investigation of the present paper. We shall henceforth call the quantum analogue of version (iii) as the weak equivalence principle of quantum mechanics (*WEQ*).

The compatibility between *WEQ* and quantum mechanics is an interesting issue which is yet to be completely settled. This issue was elaborately discussed by Greenberger[8]. Evidence supporting the violation of *WEQ* already exists in interference phenomena associated with the gravitational potential in neutron and atomic interferometry experiments[3, 4] where the observable interference patterns are

mass dependent. Further, at the theoretical level, on applying quantum mechanics to the problem of a particle bound in an external gravitational potential it is seen that the radii, frequencies and binding energy depend on the mass of the bound particle[7, 8, 9]. The possibility of quantum violation of *WEQ* is also discussed in a number of other papers, for instance using neutrino mass oscillations in a gravitational potential[10].

Recently, Davies[11] has provided a particular quantum mechanical treatment of the violation of *WEQ* for a quantum particle whose time of flight is proposed to be measured by a model quantum clock[12]. This model of quantum clock actually measures the phase change of the wave function during the particle's passage through a specified spatial region. In this treatment, Davies considered a variant of the simple Galileo experiment where particles of different mass are projected vertically in a uniform gravitational field. Quantum particles are able to tunnel into the classically forbidden region beyond the classical turning point and the tunneling depth depends on the mass. One might therefore expect a small but significant mass-dependent "quantum delay" in the return time. Such a delay would represent a violation of *WEQ*. Using the concept of the Peres clock[12] the time of flight is calculated from the *stationary state* wave function for the quantum particle moving in a gravitational potential. However, this violation is *not* found far away from the classical turning point of the particle trajectory. Within a distance of roughly one de Broglie wave length from the classical turning point there are significant quantum corrections to the turn-around time (i.e., the time taken by the particle to reach its maximum height), including the possibility of a mass-dependent delay due to the penetration of the classically forbidden region by the evanescent part of the wavefunction. Thus, this quantum "smearing" of the *WEQ* is restricted to distances within the usual position uncertainty of a quantum particle.

In another relevant gedanken experimental scheme Viola and Onofrio[13] have studied the free fall of a quantum test particle in a uniform gravitational field. Using Ehrenfest's theorem for obtaining the average time of flight for a test mass, if one takes gravitational mass to be equal to the inertial mass then the mean time taken by the particle to traverse a distance H under free fall is $\langle t\rangle = \sqrt{2H/g}$ which is exactly equal to the classical result. Viola and Onofrio made a rough estimate of the fluctuations around this mean value using a semiclassical approach with the initial wave function taken as a Schrödinger cat state. This fluctuation around the mean time of flight was shown to be dependent on the mass of the particle. However, one may note that the very definition of the time of flight or arrival time of a quantum particle is the subject of much debate, and there exists no unique or unambiguous definition that is universally applicable and also empirically well-tested[14].

As a sequel to these works by Viola and Onofrio[13] and Davies[11], we study the issue of violation of *WEQ* in the present paper from a different perspective. Note that the gravitational equivalence principle has been historically formulated at the level of single particles, which is quite appropriate within the domain of classical mechanics. However, the formulation of the quantum counterpart is experimentally verifiable only at the level of an ensemble evolving through Schrodinger dynamics. Following this line

of argument, it seems that for a quantum-classical comparison to be meaningful, even the classical results have to be stated in the framework of a distribution of particles undergoing a classical dynamical evolution[15]. To this end we consider an ensemble of identical quantum particles represented by a Gaussian wave packet which evolves under the gravitational potential. We make use of the quantum probability current in computing the mean arrival time for a wave packet under free fall. The probablility current approach[16] towards calculating the mean arrival time for an ensemble of quantum particles is conceptually sound and also well suited for our present investigation of the violation of *WEQ*.

The plan of the paper is as follows. In the next section we compute the *position detection probability* for atomic and molecular mass particles represented by a Gaussian wave packet that is projected upwards against gravity around two different points; one around the classical turning point and another around a region of the initial projection point after it returns back. We show an explicit *mass dependence* of the position probability computed around both these points, thus indicating violation of *WEQ* not only at the *turning point* of the classical trajectory, but also *far away* from it around the *initial projection point*. We then compute the mean arrival time for a wave packet under free fall in Section III. Here we consider the case when the particles are dropped from a height with zero initial velocity. We observe an explicit *mass dependence* of the *mean arrival time* at an arbitrary detector location indicating once again the manifest violation of *WEQ*. Another issue of interest as discussed by Greenberger[9] is to understand whether compatibility with *WEQ* is recovered in the macroscopic limit of quantum mechanics. We show that using the quantum probability current approach of obtaining the mean arrival time[16] of an ensemble of particles, the validity of *WEQ* emerges smoothly in the limit of large mass. We conclude with a brief summary of our results in Section IV highlighting the key differences of our approach with the earlier works.

2. Mass dependence of position detection probabilities

A beam of quantum particles with an initial Gaussian distribution is considered to be projected upwards against gravity. Subsequently, the position probability distribution is calculated within an arbitrary region either around the classical turning point of the potential $V = m_g g z$ or away from the turning point around the region from where the particles were projected. Such an observable quantity turns out to be mass dependent, as seen below.

Let us consider particles of different inertial masses that are thrown upward against gravity with the same initial mean position and mean velocity. The initial states of the quantum particles can be represented by a one dimensional Gaussian wave function given by

$$\psi^j(z, t=0) = \left(2\pi\sigma_0^2\right)^{-1/4} \exp\left(ik^j z\right) \exp\left(-\frac{z^2}{4\sigma_0^2}\right) \tag{1}$$

peaked at $z = 0$ with the initial group velocity (defined for the above wave function

evolving through the Schrodinger equation as $u = (d\omega_j)/(dk_j)$ with ω_j and k_j being the angular frequency and wave number, respectively, for the j^{th} particle) given by $u = \hbar k^j/m_i^j$, where m_i^j is the inertial mass of the j^{th} particle.

In order to perform an ideal free fall experiment for quantum particles having different inertial masses m_i^1, m_i^2,.. etc. (with suffix i representing the inertial mass, and with $m_i^1 \neq m_i^2$ etc.), one has to specify an initial preparation in such a way that any difference in the motion during the free fall must be ascribed to the effect of gravity. Now, within the classical Hamilton picture the Galileian prescription for initial positions and velocities fixes the ratio between the initial momenta in a well-defined way, i.e., $p_0^1/p_0^2 = m_i^1/m_i^2$, etc. Following Ref.[13], we extend such a prescription to the quantum case, of course keeping in mind that the Heisenberg uncertainty principle forbids the simultaneous definition of the initial position and momentum for each particle. If ψ_1 and ψ_2 denote the initial wave functions for particles 1 and 2 in the Schrödinger picture, the quantum analogue of the situation can be achieved by stipulating the conditions

$$\langle \widehat{z} \rangle_{\psi^1} = \langle \widehat{z} \rangle_{\psi^2} = 0, \qquad \frac{\langle \widehat{p}_z \rangle_{\psi^1}}{m_i^1} = \frac{\langle \widehat{p}_z \rangle_{\psi^2}}{m_i^2} \equiv u \tag{2}$$

where $\langle \widehat{z} \rangle_{\psi}$ and $\langle \widehat{p}_z \rangle_{\psi}$ denote the expectation values for position and momentum operators, respectively (confining to a one dimensional representation along the vertical z direction). The probabilistic interpretation underlying quantum mechanics allows us only to speak of probability distributions, for instance, characterized by *mean* initial conditions such as Eq.(2), as opposed to the sharply-defined values for the relevant classical observables.

With the above prescription one can consider the time evolution of the initial state under the potential $V = m_g^j g z$, where m_g^j is the gravitational mass of the j^{th} particle. At any subsequent time t the Schrödinger time evolved wave function $\psi^j(z,t)$ is given by

$$\begin{aligned}\psi^j(z,t) &= \left(2\pi s_t^2\right)^{-1/4} \exp\left[\frac{\left(z - ut + (m_g^j/m_i^j)\frac{1}{2}gt^2\right)^2}{4 s_t \sigma_0}\right] \\ &\times \ \exp\left[i(m_i^j/\hbar)\left\{\left(u - (m_g^j/m_i^j)gt\right)(z - ut/2)\right\}\right] \\ &\times \ \exp\left[i(m_i^j/\hbar)\left\{-(m_g^j/m_i^j)^2\frac{1}{6}g^2t^3\right\}\right]\end{aligned} \tag{3}$$

where $s_t = \sigma_0\left(1 + i\hbar t/2m_i^j\sigma_0^2\right)$. We see even if one takes $m_i^j = m_g^j$, i.e., equates the inertial mass with the gravitational mass, the observable position probability density $|\psi^j(z,t)|^2$ will have an explicit mass dependence

$$|\psi^j(z,t)|^2 = \left(2\pi\sigma^2\right)^{-1/2} \exp\left[-\frac{\left(z - ut + \frac{1}{2}gt^2\right)^2}{2\sigma^2}\right] \tag{4}$$

coming from the spreading of the wave packet given by $\sigma = \sigma_0\left(1 + \hbar^2t^2/4{m_i^j}^2\sigma_0^4\right)^{1/2}$ which is mass dependent.

Table 1. Mass dependence of the probability at the initial projection point. We take $u = 10^3$ cm/sec, $\sigma_0 = 10^{-3}$ cm, $\epsilon = \sigma_0$, $t = t_2 = 2u/g$ sec.

$System$	$Mass(m_i^j)$ $in(a.m.u)$	$Probability$ $P_1(m_i^j)$
H	1.00	0.0012
H_2	2.00	0.0024
Li	6.94	0.0085
Be	9.01	0.0111
C	12.01	0.0148
Ag	107.87	0.1305
C_{60}	720.00	0.5428
protein molecule	7.2×10^4	0.6826
heavier molecule	7.2×10^7	0.6826

The peak of the wave packet follows the classical trajectory and it has a turning point at the time $t = t_1 = u/g$ at $z = z_c = ut_1$. At a later time $t = t_2 = 2u/g$, when the peak of the wave packet comes back to its initial position $z = 0$, if we compute the probability of finding particles $P_1(m_i^j)$ within a very narrow region $(-\epsilon \ \ to \ \ +\epsilon)$ around this point $z = 0$ then that probability is found to be a *function of mass* and is given by

$$P_1\left(m_i^j\right) = \int_{-\epsilon}^{+\epsilon} |\psi^j(z, t_2)|^2 dz \tag{5}$$

This effect of the *mass dependence* of the probability occurs essentially because the spreading of the wave packet under gravitational potential is different for particles of different masses. We explicitly estimate this effect for different molecular mass particles. A different set of mass dependent probabilities $P_1\left(m_i^j\right)$ may be obtained by taking a different value of the width σ_0 of the initial wave packet. In the Table-1 it is shown numerically how the probability of finding the particles $P_1\left(m_i^j\right)$ around the mean initial projection point ($z = 0$) changes with the variation of mass for an initial Gaussian position distribution. We note that for further increase in mass of the particle beyond that of a protein molecule, the change in the probablity $P_1(m_i^j)$ gets negligbly small, or in other words the mass dependence of the probability gets saturated.

We then compute the probability of finding particles $P_2\left(m_i^j\right)$ at $t = t_1 = u/g$ within a very narrow detector region $(-\epsilon \ \ to \ \ +\epsilon)$ around a point which is the classical turning point $z = z_c = ut_1$ for the particle. $P_2\left(m_i^j\right)$ is also a function of mass and is given by

$$P_2(m_i^j) = \int_{-\epsilon}^{+\epsilon} |\psi^j(z, t_1)|^2 dx \tag{6}$$

In the Table-2 it is shown numerically how the probability of finding the particles $P_2(m_i^j)$ around the classical turning point changes with the variation of mass for a intial Gaussian position distribution. As in the previous case, we again find that the mass-dependence of the probablity $P_2(m_i^j)$ for finding the particle gets saturated in the limit of large mass.

Table 2. Mass dependence of the probability at the turning point. We take $u = 10^3$ cm/sec, $\sigma_0 = 10^{-3}$ cm, ϵ=σ_0, $t = t_1 = u/g$ sec.

System	$Mass(m_i^j)$ $in(a.m.u)$	$Probability$ $P_2(m_i^j)$
H	1.00	0.0024
H_2	2.00	0.0049
Li	6.94	0.0171
Be	9.01	0.0222
C	12.01	0.0296
Ag	107.87	0.2522
C_{60}	720.00	0.7277
protein molecule	7.2×10^4	0.7978
heavier molecule	7.2×10^7	0.7978

The question of the quantum-classical correspondence[17] could be elaborated further within the present context by constructing a suitable classical phase space distribution matching with the initial quantum distribution. It may be interesting to note that if one were to work with a classical ensemble of particles with an initial phase space distribution taken as the product of two Gaussian functions matching the initial position distribution $|\psi(z,0)|^2$ and its fourier transform (say, $|\phi(p,0)|^2$ representing the initial momentum distribution), essentially the same results attributed to ensemble spread are obtained through the classical Liouville evolution for Gaussian distributions[15]. Note also that within the present context the use of the Wigner function does not lead to any new insights since for the linear gravitational potential the Wigner function reproduces classical results.

3. Mass dependence of mean arrival time and the classical limit

Now let us pose the problem in a different way. We consider the quantum particle prepared in the initial state given by Eq.(1) satisfying Eq.(2) and with $u = 0$. The particle is subjected to free fall under gravity. We then ask the question as to when does the quantum particle reach a detector located at $z = Z$. In classical mechanics, a particle follows a definite trajectory; hence the time at which a particle reaches a given location is a well defined concept. On the other hand, in standard quantum mechanics, the meaning of arrival time has remained rather obscure. There exists an extensive literature on the treatment of arrival time distribution in quantum mechanics[14]. One possible internally consistent approach of defining the arrival time probability distribution is through the quantum probability current[16] which we employ in the present investigation. The probability current approach for computation of the mean arrival time of a quantum ensemble not only provides an unambiguous definition of arrival time at the quantum mechanical level[16, 18, 19], but also addresses the issue of obtaining the proper classical

limit of the time of flight of massive quantum particles[15].

It is relevant to observe here that though the Schrödinger probability current is *not* uniquely defined within nonrelativistic quantum mechanics, but for, say, particles with spin-1/2, it has been shown by Holland[18] by taking the nonrelativistic limit of the Dirac probability current, that the quantum probability current contains a term that is spin dependent. The arrival time distribution is then uniquely formulated using the probability current obtained by taking the nonrelativistic limit of the corresponding relativistic current. It was shown using the explicit example of a Gaussian wave packet that the spin-dependence of the probability current leads to the spin-dependence of the mean arrival time for free particles[19]. However, for the case of massive spin-0 particles it has been shown recently by taking the non-relativistic limit of Kemmer equation[20] that the unique probability current is given by the Schrödinger current[21]. Hence, the Schrödinger probability current density can be used to define a precise and logically consistent arrival time distribution for spin-0 quantum particles, that is relevant for the present analysis.

The expression for the Schrödinger probability current density $J(Z,t)$ at the detector location $z = Z$ for the time evolved state is calculated using the initial state prepared in the Gaussian form given by Eq.(1) and satisfying Eq.(2). The particle falls freely under gravity along $-\widehat{z}$ direction from the initial peak position at $z = 0$ with $u = 0$ and $J(Z,t)$ is given by

$$J(Z,t) = \rho(Z,t)\ \ v(Z,t) \tag{7}$$

where

$$\rho(Z,t) = (2\pi\sigma^2)^{-1/2}\, exp\left[-\frac{(Z-\frac{1}{2}gt^2)^2}{2\sigma^2}\right] \tag{8}$$

and

$$v(Z,t) = \left[gt + \frac{\hbar^2 t}{4{m_i^j}^2\sigma_0^2\sigma^2}(Z - gt^2/2)\right] \tag{9}$$

Taking the modulus of the probability current density as determining the arrival time distribution[16], the mean arrival time $\overline{\tau}$ at a particular detector location is computed for an ensemble of particles with an initial Gaussian position distribution *falling freely* under gravity. Then this observable quantity $\overline{\tau}$ is given by

$$\overline{\tau}\left(m_i^j\right) = \frac{\int_0^\infty |J\,(Z,t)|\, t\, dt}{\int_0^\infty |J\,(Z,t)|\, dt} \tag{10}$$

which is actually the first temporal moment of the modulus of the probability current density. Since $\sigma = |s_t| = \sigma_0\left(1 + \hbar^2 t^2/4{m_i^j}^2\sigma_0^4\right)^{1/2}$ is mass dependent, it is seen from Eqs.(7–9) that $J\,(Z,t)$ is mass-dependent too. Hence the mean arrival time $\overline{\tau}$ calculated by using Eq.(10) for the Gaussian wave packets corresponding to different atomic mass particles falling freely under gravity is also mass dependent.

In FIG.1, we depict the variation with mass of the mean arrival time at a particular detector location for an ensemble of particles under free fall. The initial conditions

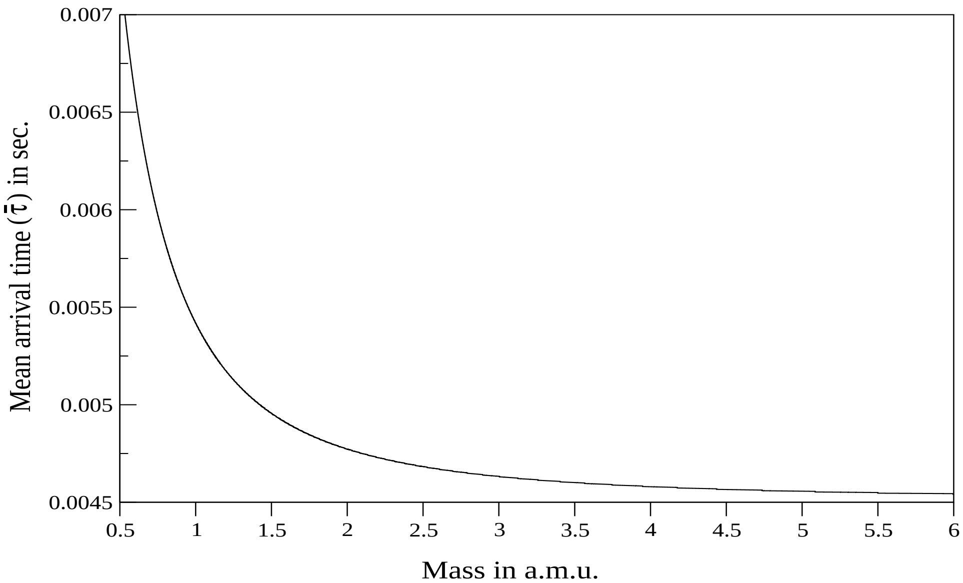

Figure 1. The variation of mean arrival time with mass (in atomic mass unit) at a detector location Z for an initial Gausssian position distribution. We take $\sigma_0 = 10^{-4}\ \ cm$, $Z = 10^{-2}\ \ cm$.

are taken as $\langle z \rangle_0 = 0$ and $\langle p \rangle_0 = 0$, where $\langle z \rangle_0$ and $\langle p \rangle_0$ are the position and momentum expectation values at $t = 0$. One should note that though the integral in the numerator of Eq.(10) formally diverges, several techniques have been employed in the literature ensuring rapid fall off for the probability distributions asymptotically[22], so that convergent results are obtained for the integrated arrival time. For our present purposes it is sufficient to employ the simple strategy of taking a cut-off ($t = T$) in the upper limit of the time integral with $T = \sqrt{2(Z + 3\sigma_T)/g}$ where σ_T is the width of the wave packet at time T. Thus, our computations of the arrival time are valid up to the 3σ level of spread in the wave function.

One can see from FIG.1 that in the limit of large mass the mean arrival time $\overline{\tau}$ asymptotically approaches the classical result which is mass independent. As was discussed by Greenberger[9], the question as to whether compatibility of the weak equivalence principle with quantum mechanics emerges in the classical limit is clouded by conceptual intricacies of obtaining the proper macroscopic limit of quantum mechanics. We see here again the probability current approach offers an effective and consistent scheme for obtaining the macroscopic limit of the arrival time distribution by continuously increasing the mass of the particle. We find that the classical value of mean arrival time is obtained as the mass dependence vanishes in the limit of large mass. We are thus able to show that compatibility of the weak equivalence principle with quantum mechanics emerges in a smooth manner in the macroscopic limit.

4. Summary and conclusions

To summarize, we have revisited a gedanken version of the quantum analogue of Galileo's leaning tower experiment with atomic and molecular mass wave packets falling freely under gravity. Our results of mass-dependence of the position detection probabilities and the arrival time distribution clearly indicate the manifest violation of the quantum analogue [5] of the weak equivalence principle (*WEQ*) stated earler. Davies[11] provided a particular quantum mechanical treatment of the violation of *WEQ* using the concept of the Peres clock[12] where the time of flight is calculated from the stationary state wave function for the quantum particle moving in a gravitational potential. However, this violation was *not* found far away from the classical turning point of the particle trajectory and was restricted to distances within the usual position uncertainty of the quantum particle. A semi-classical approach based on the Ehrenfest theorem yields the classical result for the average time of flight and mass dependence for fluctuations around the average[13]. Our approach, on the other hand, is based on the quantum probability current approach and leads to the mass dependence of the arrival time distribution computed around any position along the trajectory of the particles. The predicted violation of *WEQ* in this case is, in principle, observable for molecular mass particles.

We have further discussed the issue of compatibility of *WEQ* with the macroscopic limit of quantum mechanics[9]. For this purpose it is essential to consider the evolution of an ensemble of particles that we have done using a Gaussian wave packet. We see that the variation of the detection probability with mass disappears in the limit of large mass of the freely falling particles, as is expected for classical objects. This saturation of the detection probability is also reflected in the mean arrival time defined through the quantum probability current, which approaches the classical result in a continuous manner with the increase of mass. We have seen that the compatibility of *WEQ* with quantum mechanics can be restored in the classical limit within this framework for particles falling freely under gravity. Our analysis has been carried out using a minimum uncertainty Gaussian wave packet. Following our approach, it should be interesting to investigate the issue of compatibility of the weak equivalence principle with quantum mechanics in the macroscopic limit for other types of Gaussian and non-Gaussian wave packets.

Finally, we would like to re-emphasize that our approach of demonstrating the quantum violation of the weak equivalence principle is different from that of other examples in that using our scheme it should be possible to predict the specific mass range of molecules where an explicit violation of *WEQ* may occur either through the measurement of the position detection probabilities, or through the mean arrival time. Our approach is capable of providing a precise prediction of the quantum violation of the weak equivalence principle in the relevant mass ranges as one goes from the micro to macro limit, and is thus amenable to experimental verification, thereby complementing other works probing the transition between the quantum and the classical domains[23].

We conclude by stressing that it should be worthwhile to compute the results in our example using other approaches[14] to calculate the quantum arrival time distribution, and compare such results with those of the present paper. Such studies can further motivate the formulation of actual experiments to decide which particular approach is empirically tenable for description of the arrival time distribution of quanta in the gravitational potential.

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